

Plan

Review of Cycle
NB-LDPC Codes

Motivation
Optimization

Split NB-LDPC
Codes

Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Generalized NonBinary LDPC Codes: Split-LDPC Codes

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Plan

Review of Cycle
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Motivation
Optimization

Split NB-LDPC
Codes

Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Non-binary Cycle LDPC Codes

Non Binary Parity Check on the i^{th} row

$$\sum_{j=1}^{d_c} h_{ij} \cdot x_j = 0 \quad \text{in } \mathbb{F}_{2^p}$$

$$\mathbb{F}_{2^p} = \{0, \alpha^0, \alpha^1, \dots, \alpha^{2^p-1}\}$$

we limit the study of the codes to the case of :

- *ultra-sparse* Tanner graphs,
- *cycle graphs*,
- constant connexion degree $d_v = 2$ on symbol nodes.

Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Why ultra-sparse $d_v = 2$ Tanner Graphs

- Amazing girths can be obtained for small to moderate codeword lengths

	$d_c = 3$	$d_c = 4$	$d_c = 6$	$d_c = 8$
$N_b = 500$	$g=22$	$g=18$	$g=14$	$g=8$
$N_b = 3000$	$g=28$	$g=20$	$g=16$	$g=12$

- Number of independant decoding iterations = $\lfloor g/4 \rfloor$. If $g \uparrow$, message passing decoder is closer to MLD,
- Non-binary codes with $d_v = 2$ in $\mathbb{F}(q)$ approach random coding bounds as $q \uparrow$ [Hu,2004]

[Hu,2004] X.Y. HU AND E. ELEFThERIOU, "BINARY REPRESENTATION OF CYCLE TANNER-GRAPH GF(2^q) CODES", *IEEE Int. Conf. on Commun.*, PARIS, FRANCE, JUNE 2004

Plan

Review of Cycle
NB-LDPC Codes

Motivation
Optimization

Split NB-LDPC
Codes

Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Finite Length Optimization of NB-LDPC Codes

Observation

*The low-weight codewords of a cycle NB-LDPC code are concentrated in **Cycles** and **Stopping Set** Topologies*

- 1 Design of Tanner Graph** : one should aim at [\[Venkiah,2008\]](#)
 - **maximum** possible girth,
 - **minimum** number of “small” cycles,
 - **minimum** the number of stopping sets build from “small” cycles.
- 2 Choice of Nonzeros values** on the edges of the small cycles and the small stopping sets in order to improve the *local minimum distance* [\[Poulliat,2008\]](#).

[\[VENKIAH,2008\]](#) A. VENKIAH ET AL., “DESIGN OF CAGES WITH A RANDOMIZED PROGRESSIVE EDGE GROWTH ALGORITHM”, *IEEE Commun. Letters*, APRIL 2008.

[\[POULLIAT,2008\]](#) C. POULLIAT ET AL., “DESIGN OF REGULAR $(2, d_c)$ -LDPC CODES OVER $GF(q)$ USING THEIR BINARY IMAGES”, *IEEE Trans. on Commun.*, OCTOBER 2008.



Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

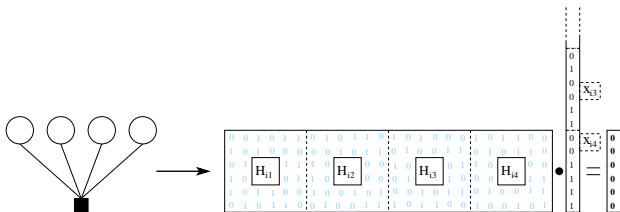
Conclusion

Concept of Binary Component Code

- Binary vector interpretation of a NB Parity-check :

$$\sum_{j=1}^{d_c} H_{ij} \cdot \underline{x}_j = \underline{0}_p \quad \text{in } (\mathbb{F}_2)^p$$

- \underline{x}_j is the binary map of length p bits,
- H_{ij} is the $(p \times p)$ binary companion matrix of some α^k .



Plan

 Review of Cycle
NB-LDPC Codes

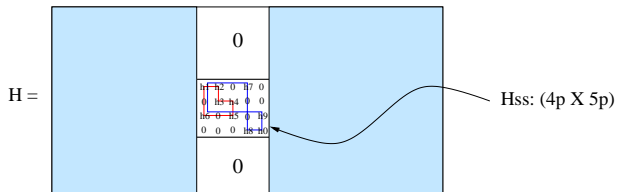
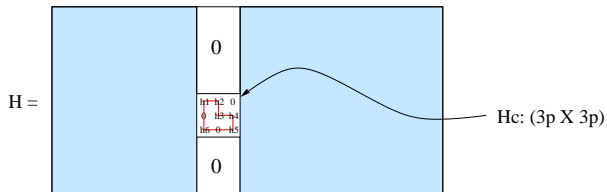
 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Concept of Binary Sub-code



remark : length(topology) = number of symbols included in a closed topology

Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Design Procedure

- 1** List all Cycles of length $\in \{g/2, g/2 + 1, \dots, g/2 + \gamma_1\}$ \mathcal{S}_c
 List all Stopping Sets of length
 $\in \{\lfloor 3g/4 \rfloor, \lfloor 3g/4 \rfloor + 1, \dots, \lfloor 3g/4 \rfloor + \gamma_2\}$ \mathcal{S}_{ss}
- 2** Consider a single set of d_c Field values as component code
 $[\alpha^1, \dots, \alpha^{d_c}]$
- 3** **Random walk** Optimization (one iteration) :
 for each and every row : replace $[\alpha^1, \dots, \alpha^{d_c}]$ by $[\alpha^{\pi_1}, \dots, \alpha^{\pi_{d_c}}]$ *iff*

$$Cost([\alpha^{\pi_1}, \dots, \alpha^{\pi_{d_c}}]) < Cost([\alpha^1, \dots, \alpha^{d_c}])$$

where $Cost(\cdot)$ is a weighted objective function of the low-weight spectrum computed on $\mathcal{S}_c \cup \mathcal{S}_{ss}$. $Cost(\cdot)$ decreases if the low-weight spectrum improves.

Plan

 Review of Cycle
NB-LDPC Codes

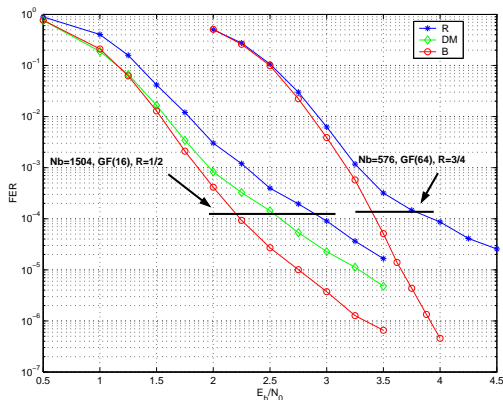
 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Results with Finite Length Optimization



PB : Error floor is still too high for $d_v = 2$ LDPC codes, and error floor is caused by low-weight codewords (undetected errors).

Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Split NB-LDPC Codes

Question

Can we Improve the Global Minimum Distance of the Code with Cycle Graphs

1 0 1 0 1 1 0 0 0 0 0 0					
0 1 0 1 0 0 0 0 0 0 0 0					
0 0 1 1 0 1 0 0 0 0 0 0				0	
1 0 0 1 1 1 0 0 0 0 0 0					
1 0 0 0 1 0 0 0 0 0 0 0					
0 1 0 1 0 1 0 0 0 0 0 0					
			0	0	
0					0

 Classical NB-LDPC
Code on Field

- All zeros clusters
 Full-rank nonzeros clusters

1 0 1 0 0 0 0 0								
0 1 0 1 0 0 0 0								
0 0 1 1 0 0 0 0				0				0
1 0 0 1 0 0 0 0								
1 0 0 0 0 0 0 0								
0 1 0 1 0 0 0 0								
			0		0	0		
0				0				0

 Generalized Split
LDPC Code

Plan

Review of Cycle
 NB-LDPC Codes
 Motivation
 Optimization

Split NB-LDPC
 Codes
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 Frame Error Rate
 Simulations

Conclusion

Definition of Split NB-LDPC codes

Generalized Parity Check Equation

$$\sum_{j=1}^{d_c} h_{ij}(x_j) = 0 \quad \text{in } \mathbb{G}(2^{p_2})$$

where $h_{ij}(\cdot) : \mathbb{G}(2^{p_{1j}}) \rightarrow \mathbb{G}(2^{p_2})$

$$\sum_{j=1}^{d_c} H_{ij} \cdot \underline{x}_j = \underline{0}_{p_2} \quad \text{in } (\mathbb{F}_2)^{p_2}$$

H_{ij} is a binary matrix ($p_{1j} \times p_2$) of full column rank p_{1j}
 called **Binary Cluster**

Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

 Split NB-LDPC
Codes

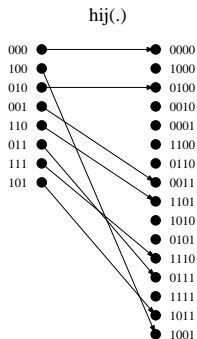
Definition and Structure

 Distance spectrum
Frame Error Rate
Simulations

Conclusion

Structure of functions $h_{ij}(\cdot)$

$$H_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$



Iterative Message Passing Decoder still follows BP equations
[Goupil,2007]

[GOUPIL,2007] A. GOUPIL ET AL., "FFT-BASED BP DECODING OF GENERAL LDPC CODES OVER ABELIAN GROUPS," *IEEE Trans. on Commun.*, APRIL 2007.

Plan

Review of Cycle
NB-LDPC Codes

Motivation
Optimization

Split NB-LDPC
Codes

Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Existing Codes as Particular Cases

- 1 if $p_{1_j} = p_2 \forall j$, gives a non binary LDPC code. Field code $\mathbb{F}_{2^{p_2}}$ if clusters are companion matrices [Poulliat'2008,Pfletschinger'2009], and General Linear Group $\mathbb{G}(2^{p_2})$ if not [Rathi'2005,Chen'2009].
- 2 if $p_{1_j} = 1 \forall j$, reduces clusters to columns vectors of length p_2 bits, and gives the structure of Generalized Low-Density Codes (GLD codes) [Boutros'1999].
- 3 if $p_{1_j} = p_2/2 \forall j$ and component code is defined on a field [Voicila'2008].

Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

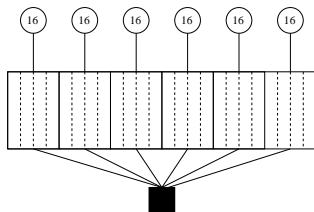
 Split NB-LDPC
Codes

Definition and Structure

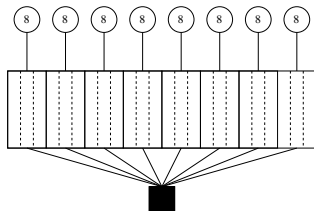
 Distance spectrum
Frame Error Rate
Simulations

Conclusion

Tanner Graphs of the Considered Split-Codes ($R = 1/2$ in $\mathbb{G}(64)$)


Type C₂

 Tanner Graph with $dc=6$

 100% sub-symbols in $\mathbb{G}(16)$

Type C₃

 Tanner Graph with $dc=8$

 100% sub-symbols in $\mathbb{G}(8)$

Plan

Review of Cycle
NB-LDPC Codes

Motivation
Optimization

Split NB-LDPC
Codes

Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Choice of Component Code

- for $R = 1/2$, in $\mathbb{G}(64)$, size of CC is $H_{cc} : 6 \times 24$ bits,
 - e-Hamming code ($N = 32, K = 26, D_{min} = 4$),
 - H_{ij} are composed of a selection of p_{1j} columns of the e-Hamming code,
- for $R = 3/4$, in $\mathbb{G}(64)$, size of CC is $H_{cc} : 6 \times 48$ bits,
 - Hamming code ($N = 63, K = 57, D_{min} = 3$),
 - H_{ij} are composed of a selection of p_{1j} columns of the Hamming code,

Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Distance Spectrum Results : Rate $R = \frac{1}{2}$

	$N_b = 576$ bits	$N_b = 2304$ bits
Field GF(64)	weight 19 → #1 weight 20 → #54 weight 21 → #258 weight 22 → #668 weight 23 → #1474 weight 24 → #3167	weight 24 → #1 weight 25 → #15 weight 26 → #167 weight 27 → #489 weight 28 → #966 weight 29 → #1738
Split C_2 GF(64)	weight 20 → #2 weight 21 → #5 weight 22 → #13 weight 23 → #25 weight 24 → #59	weight 26 → #5 weight 28 → #10
Split C_3 GF(64)	weight 19 → #2 weight 22 → #1 weight 23 → #2 weight 24 → #3	weight 34 → #1 weight 55 → #1 weight 56 → #1 weight 59 → #1

Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum

 Frame Error Rate
Simulations

Conclusion

Distance Spectrum Results : Rate $R = \frac{3}{4}$

	$N_b = 576$ bits	$N_b = 2304$ bits
Field GF(64)	weight 9 → #4 weight 10 → #137 weight 11 → #1071 weight 12 → #4851	weight 11 → #1 weight 12 → #20 weight 13 → #423 weight 14 → #1903 weight 15 → #6493
Split C_2 GF(64)	weight 10 → #13 weight 11 → #106 weight 12 → #503	weight 11 → #1 weight 12 → #1 weight 13 → #11 weight 14 → #108 weight 15 → #335
Split C_3 GF(64)	weight 9 → #10 weight 10 → #29 weight 11 → #122 weight 12 → #521	weight 12 → #4 weight 13 → #11 weight 14 → #47 weight 15 → #98

Plan

 Review of Cycle
NB-LDPC Codes

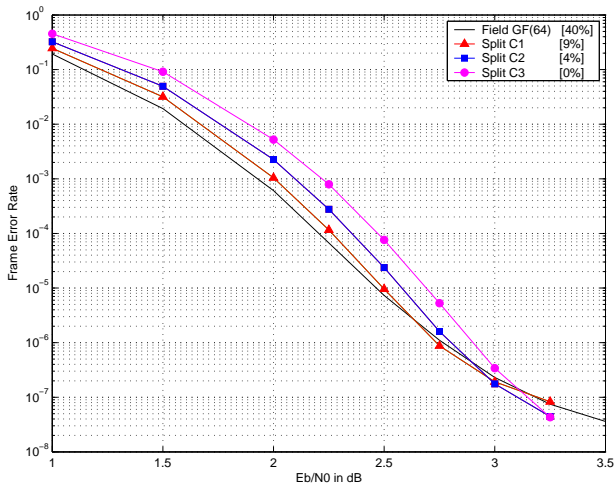
 Motivation
Optimization

 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum
Frame Error Rate
Simulations

Conclusion

Rate $R = \frac{1}{2}$ code, $N_b = 576$, 100 iter. of BP



Plan

 Review of Cycle
NB-LDPC Codes

 Motivation
Optimization

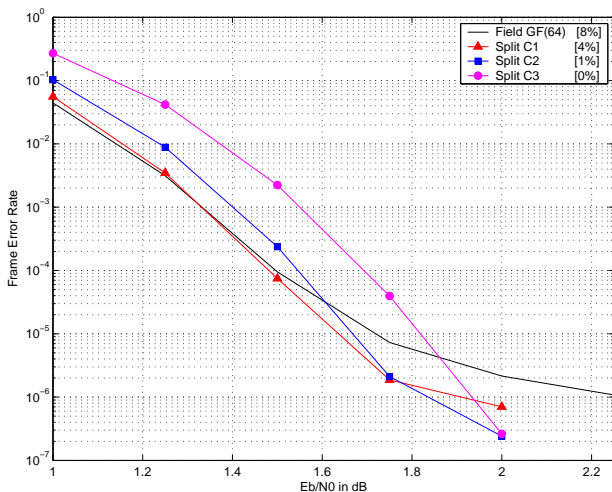
 Split NB-LDPC
Codes

 Definition and Structure
Distance spectrum

 Frame Error Rate
Simulations

Conclusion

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Plan

 Review of Cycle
 NB-LDPC Codes

 Motivation
 Optimization

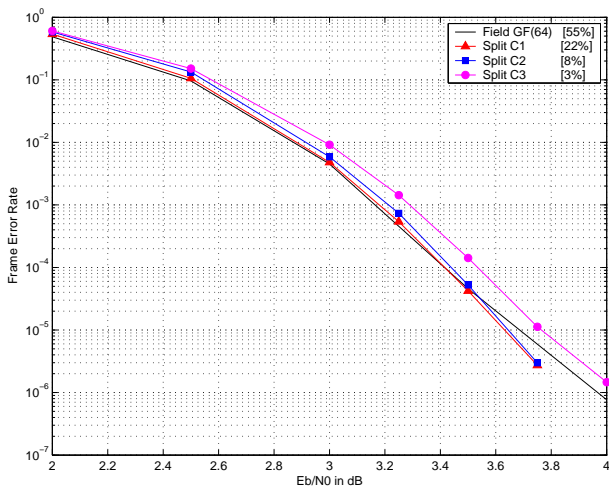
 Split NB-LDPC
 Codes

 Definition and Structure
 Distance spectrum

 Frame Error Rate
 Simulations

Conclusion

Rate $R = \frac{3}{4}$ code, $N_b = 576$, 100 iter. of BP



Plan

Review of Cycle
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Optimization

Advantages

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Codes

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Simulations

Conclusion

Drawbacks

Conclusion

- Slight gain in the error floor region compared to field codes,
- Important improvement of the code spectrum : ↓ undetected errors,
- messages are stored at the *sub-symbol* level : less memory required for hardware implementation.
- Slight degradation of the waterfall,
- Extra decoding complexity due to denser graph,
- Code design is more difficult than for field codes.