Randomized Space-Time Block Codes for Cooperative Communications: Asymptotic Performance Analysis

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Outline

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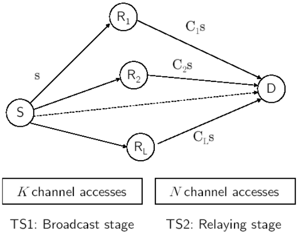
   - Motivation
   - Known results
2. Outage Results
   - Preliminary concepts
   - Amplify and Forward
   - Decode and Forward
3. Conclusions

Motivation



- We consider a simple relay communication system where a set of L relays aids a point-to-point communication.
- Relays employ random linear space-time block coding (STBC) to introduce diversity.
- Two transmission phases:
  1. the source broadcasts K symbols;
  2. relays encode the K source symbols into N new ones to be forwarded.

  The destination estimates the information from the signals received in both phases.

- To design classic STC's, the maximum number of transmitters must be known.
- STC-design complexity increases with the number of transmitters.

Motivation (Cont'd)

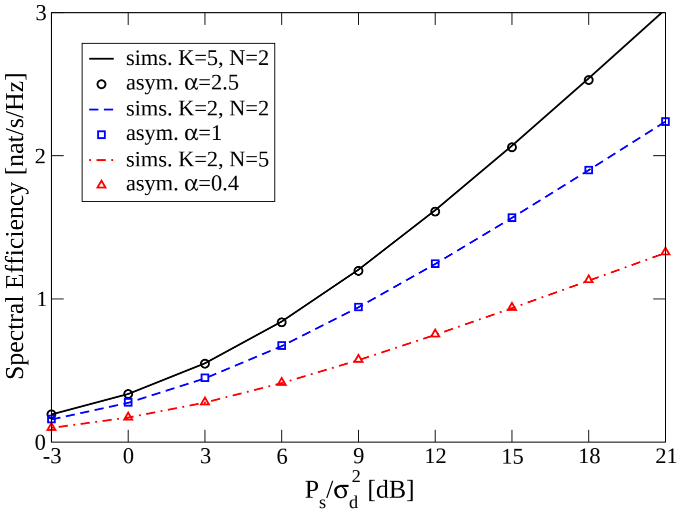
Why random STBC?

- To design classic STC's, the maximum number of transmitters must be known.
- STC-design complexity increases with the number of transmitters.

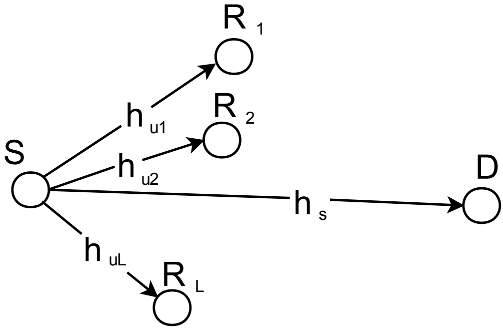
Random STBC's:

- do not present design issues;
- are flexible in the number of active relays;
- multiuser-detection results suggest random STBC's are robust to asynchronous transmissions.

- The elements of the STBC matrices are drawn from a random process with zero mean and variance $1/N$.
- For $K, N \to +\infty$ and $K/N \to \alpha$, system metrics (e.g. the spectral efficiency) do not depend on any code statistics but only on the ratio α.
- Asymptotic results are very representative of the finite reality, even for very low K, N. Note $\alpha = K/N$ is the ratio between source and relay transmitting times.


3
2
1
0
sims. K=5, N=2
asym. α=2.5
sims. K=2, N=2
asym. α=1
sims. K=2, N=5
asym. α=0.4
Spectral Efficiency [nat/s/Hz]
-3
0
3
6
9
12
15
18
21
P_s/σ_d^2 [dB]

In the first time-slot, the source broadcasts its information to the destination (through h_s) and the L relays (through $\{h_{ul}\}_{l=1}^{L}$).

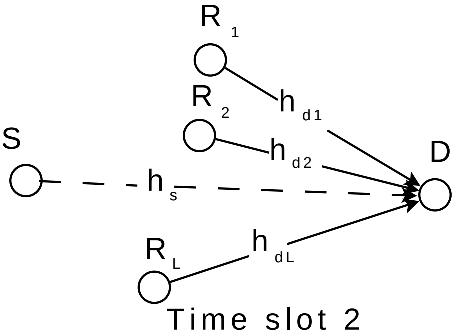


The destination receives:

- first time-slot

$$\mathbf{d}_1 = h_s \mathbf{s} + \mathbf{n}_1;$$

In the second time-slot, the relays broadcast the source information to the destination (through $\{h_{dl}\}_{l=1}^{L}$).



The destination receives:

- first time-slot

$$\mathbf{d}_1 = h_s \mathbf{s} + \mathbf{n}_1;$$

- second time-slot

$$\mathbf{d}_2 = \begin{cases} \left(\sum_{l=1}^{L} g_l h_{dl} h_{ul} \mathbf{C}_l \right) \mathbf{s} + \sum_{l=1}^{L} g_l h_{dl} \mathbf{C}_l \mathbf{n}_{ul} + \mathbf{n}_d & \text{AF,} \\ \left(\sum_{l \in \mathcal{L}'} g_l h_{dl} \mathbf{C}_l \right) \mathbf{s} + \mathbf{n}_d & \text{DF,} \end{cases}$$

where $\mathcal{L}' \subseteq \mathcal{L} = \{1, \ldots, L\}$ is the decoding subset.

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  where $\mathcal{L}' \subseteq \mathcal{L} = \{1, \ldots, L\}$ is the decoding subset.

- Assuming full CSIR, the spectral efficiency has been computed for two different receivers: the optimal ML and the linear MMSE.

With the optimal ML receiver, the maximum information is recovered from the signal received by the destination. For the presented, MIMO-like signal model, the maximum spectral efficiency is

$$I_{AF,OPT} = \frac{1}{N+K} \ln\det\Bigg(\mathbf{I}_K + \\ + P_s \begin{bmatrix} h_s \mathbf{I}_K \\ \widetilde{\mathbf{C}}\widetilde{\mathbf{\Psi}}\widetilde{\mathbf{H}}_u \end{bmatrix}^H \begin{bmatrix} \sigma_d^2 \mathbf{I}_K & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}^{-1} \begin{bmatrix} h_s \mathbf{I}_K \\ \widetilde{\mathbf{C}}\widetilde{\mathbf{\Psi}}\widetilde{\mathbf{H}}_u \end{bmatrix} \Bigg),$$

where $\mathbf{R} = \sigma_u^2 \widetilde{\mathbf{C}}\widetilde{\mathbf{\Psi}}\widetilde{\mathbf{\Psi}}^H \widetilde{\mathbf{C}}^H + \sigma_d^2 \mathbf{I}_N$.

The dependence on the specific realization of the spreading matrices $\widetilde{\mathbf{C}}$ can be removed in the asymptotic domain.

For the ML receiver, the asymptotic equivalent of the spectral efficiency is [IEEE Trans. Wireless Commun.]:

$$I_{AF,OPT}^{(as)} = \frac{1}{1+\alpha}\left[\alpha \ln\left(1 + \frac{P_s}{\sigma_d^2}|h_s|^2\right) + \right.$$
$$\left. + \alpha \sum_{l=1}^{L} \ln \frac{1+\lambda_l \phi_1}{1+|g_l h_{dl}|^2 \phi_2} + \ln \frac{\phi_2}{\phi_1} + \frac{\sigma_d^2}{\sigma_u^2}(\phi_1 - \phi_2)\right],$$

where ϕ_1, ϕ_2 are the unique positive solutions to

$$\phi_1 = \left(\frac{\sigma_d^2}{\sigma_u^2} + \alpha \sum_{l=1}^{L} \frac{\lambda_l}{1+\lambda_l \phi_1}\right)^{-1}, \quad \phi_2 = \left(\frac{\sigma_d^2}{\sigma_u^2} + \alpha \sum_{l=1}^{L} \frac{|g_l h_{dl}|^2}{1+|g_l h_{dl}|^2 \phi_2}\right)^{-1}$$

and $\{\lambda_l\}_{l=1}^{L}$ are the solutions to

$$\frac{P_s/\sigma_u^2}{1+P_s|h_s|^2/\sigma_d^2} \sum_{l=1}^{L} \frac{|g_l h_{dl} h_{ul}|^2}{\lambda - |g_l h_{dl}|^2} = 1.$$

The LMMSE receiver is the best (in terms of SNIR) linear receiver. The spectral efficiency is given by the Shannon formula $I = \frac{K}{K+N} \ln(1 + SNIR)$, since $SNIR$ is the same for all the transmitted symbols.

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$$I_{AF,LMMSE}^{(as)} = \frac{\alpha}{1+\alpha} \ln \left(1 + \underbrace{\frac{P_s}{\sigma_d^2} |h_s|^2 + \frac{P_s}{\sigma_u^2} \sum_{l=1}^{L} \frac{|g_l h_{dl} h_{ul}|^2}{|g_l h_{dl}|^2 + \frac{1}{\phi_1}}}_{\text{signal-to-noise-plus-interference ratio}} \right),$$

with ϕ_1 as before.

The dependence on the specific instance of the signatures has been removed also in this case.

The DF case

Given the decoding set $\mathcal{L}'$ and defined

$$\beta_{\mathcal{L}'} = 1 + \alpha|\mathcal{L}'|\beta_{\mathcal{L}'} \frac{\frac{P_s/\sigma_d^2}{1+P_s|h_s|^2/\sigma_d^2} \sum_{l\in\mathcal{L}'} |h_{dl}|^2}{\beta_{\mathcal{L}'} + \frac{P_s/\sigma_d^2}{1+P_s|h_s|^2/\sigma_d^2} \sum_{l\in\mathcal{L}'} |h_{dl}|^2},$$

the asymptotic equivalents of the spectral efficiencies are [IEEE ISIT 2009]:

$$I_{DF,OPT}^{(\mathcal{L}')} = \frac{1}{1+\alpha}\left[\alpha \ln\left(1 + \frac{P_s}{\sigma_d^2}|h_s|^2\right) + \ln\beta_{\mathcal{L}'} + \frac{1}{\beta_{\mathcal{L}'}} + \right.$$
$$\left. + \alpha \sum_{l\in\mathcal{L}'} \ln\left(1 + \frac{P_s/\sigma_d^2}{1+P_s|h_s|^2/\sigma_d^2}\frac{|h_{dl}|^2}{\beta_{\mathcal{L}'}}\right) - 1\right]$$

$$I_{DF,LMMSE}^{(\mathcal{L}')} = \frac{\alpha}{1+\alpha}\ln\left(1 + \frac{P_s}{\sigma_d^2}|h_s|^2 + \frac{P_s}{\sigma_d^2\beta_{\mathcal{L}'}}\sum_{l\in\mathcal{L}'}|h_{dl}|^2\right)$$

Outage and diversity

Assumptions:

- Frequency flat, quasi-static channel coefficients.
- Rayleigh fading

Given a target rate R, the outage probability is defined as

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Definition

The outage gain κ and the diversity order d are defined such that

$$\lim_{SNR \to +\infty} SNR^d P_{out}(R) = \kappa.$$

Assuming $R = r \ln SNR$, then

$$d(r) = -\lim_{SNR \to +\infty} \frac{\ln P_{out}(r \ln SNR)}{\ln SNR}$$

expresses the tradeoff between diversity order d and multiplexing gain r.

For the aforementioned L-relay system, the diversity order obtained with the ML receiver is [EUSIPCO 2009]:

$$d = \begin{cases} L+1 & \text{for } \alpha < \dfrac{1}{L-1}, \\ M+1 & \text{for } \dfrac{1}{M} \leq \alpha < \dfrac{1}{M-1}, \end{cases}$$

with $1 \leq M < L$.

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1. The spreading matrices $\mathbf{C}_l$ have to be tall $(N > (L-1)K)$ to minimize the outage probability. Note, however, that this implies a sensible waste of degrees of freedom, since the source is transmitting K symbols every $K + N$ channel accesses;

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2. Relaying is always superior to the direct link, which only achieves a unitary diversity order.

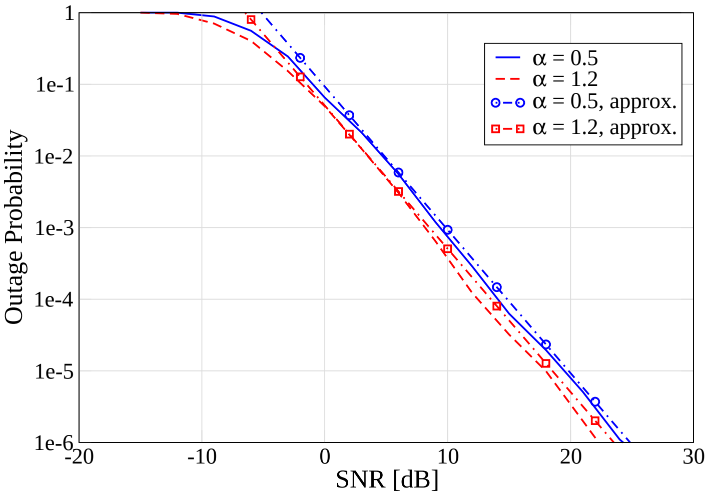
With one single relay, the diversity order is always $d = 2$ and the outage gain is

$$\kappa = \frac{\zeta_s \zeta_u}{z} \int_{\mathbb{R}_+^2} \mathbb{1}\{\mathcal{E}_1\} \,\mathrm{d}a \,\mathrm{d}b + \frac{\zeta_s \zeta_d}{z} \int_{\mathbb{R}_+^2} \mathbb{1}\{\mathcal{E}_2\} \,\mathrm{d}a \,\mathrm{d}c,$$

with

$$\mathcal{E}_1 : \alpha \ln(1 + a) + \min(1, \alpha) \ln\left(1 + \frac{b}{1 + a}\right) < (1 + \alpha) R$$

$$\mathcal{E}_2 : c < \frac{\alpha z (1 + a)}{\gamma \left(\frac{\alpha}{1 - \gamma} - 1\right)}.$$


α = 0.5
α = 1.2
α = 0.5, approx.
α = 1.2, approx.
Outage Probability
1
1e-1
1e-2
1e-3
1e-4
1e-5
1e-6
-20
-10
0
10
20
30
SNR [dB]

For the single-relay case, we have studied also the diversity achieved by the LMMSE receiver [IEEE ICC 2009]:

- for $\alpha \leq 1$, $d = 2$ and $\kappa = \kappa_1$;
- for $1 < \alpha \leq \alpha_{th}$, $d = 2$ and $\kappa = \kappa_2$;
- for $\alpha > \alpha_{th}$, $d = 1$ and $\kappa = \kappa_3$,

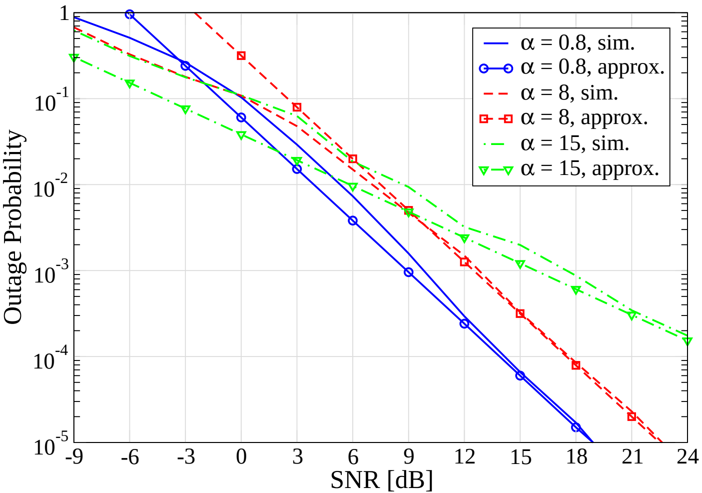
where

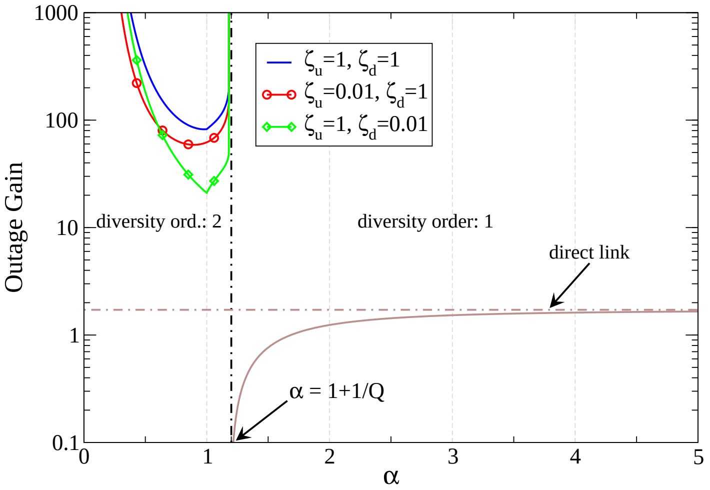
$$\alpha_{th} = 1 + 1/Q(\alpha_{th}), \qquad Q(\alpha) = \exp\left(\frac{1+\alpha}{\alpha} R\right) - 1.$$

$$\kappa_1 = \frac{\zeta_s \zeta_u}{z} \frac{Q^2}{2} + \zeta_s \zeta_d (Q+1) \left[-Q - \frac{Q+1}{\alpha} \ln\left(1 - \alpha \frac{Q}{Q+1}\right)\right];$$

$$\kappa_2 = \frac{\zeta_s \zeta_u}{z} \left\{ \frac{Q^2}{2} + \frac{\alpha - 1}{\alpha}(Q+1) \left[-Q - \frac{Q+1}{\alpha} \ln\left(1 - \alpha \frac{Q}{Q+1}\right)\right]\right\} + \\ + \zeta_s \zeta_d (Q+1) \left[-Q - \frac{Q+1}{\alpha} \ln\left(1 - \alpha \frac{Q}{Q+1}\right)\right];$$

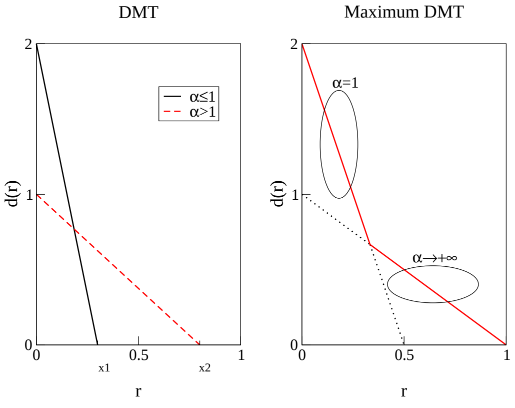
$$\kappa_3 = \zeta_s \frac{(\alpha - 1)Q - 1}{\alpha}.$$


α = 0.8, sim.
α = 0.8, approx.
α = 8, sim.
α = 8, approx.
α = 15, sim.
α = 15, approx.
Outage Probability
1
10^-1
10^-2
10^-3
10^-4
10^-5
-9
-6
-3
0
3
6
9
12
15
18
21
24
SNR [dB]


1000
100
10
1
0.1
0
1
2
3
4
5
α
Outage Gain
ζu=1, ζd=1
ζu=0.01, ζd=1
ζu=1, ζd=0.01
diversity ord.: 2
diversity order: 1
direct link
α = 1+1/Q

In this case the DMT is as follows:

$$d(r) = \begin{cases} 2\left(r\dfrac{1+\alpha}{\alpha} - 1\right) & \text{for } \alpha \leq 1; \\ r\dfrac{1+\alpha}{\alpha} - 1 & \text{for } \alpha > 1. \end{cases}$$



The decoding set has to be taken into account by means of the total probability theorem:

$$P_{out}(R) = \sum_{\mathcal{L}' \subseteq \mathcal{L}} \Pr[\mathcal{L}' \text{ is the decoding set}] \Pr[I^{as}_{OPT,\mathcal{L}'} < R].$$

It can be shown [IEEE ISIT 2009] that the ML receiver always achieves full diversity order $d = L + 1$ and the outage gain can be computed numerically.

This case has been analyzed in [IEEE ISIT 2009].

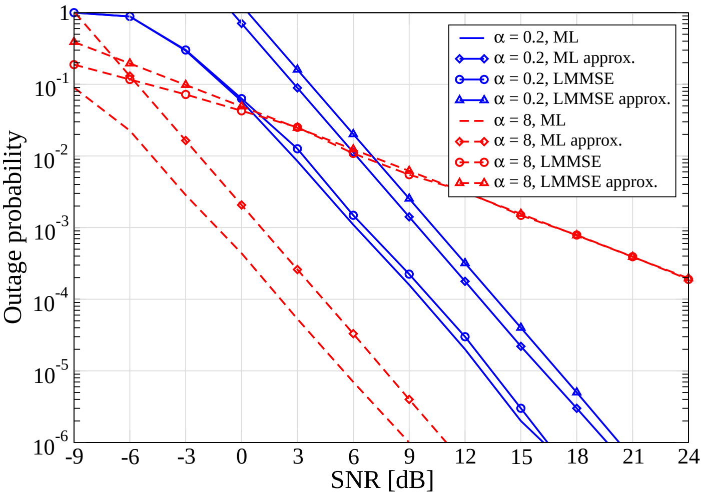
- For $\alpha \leq \alpha_{th}$, the LMMSE receiver achieves full diversity order $d = L + 1$.
- For $\alpha > \alpha_{th}$, the LMMSE receiver achieves diversity order $d = 1$.

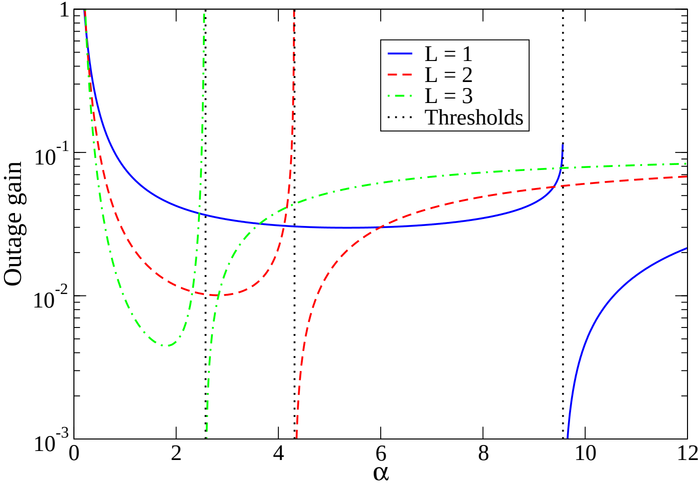
The outage gain can be computed in closed form. For example, for the 1-relay case [ICT MobileSummit 2009]

$$\kappa = \begin{cases} \frac{\zeta_s \zeta_u}{z} Q^2 + \zeta_s \zeta_d (Q+1) \left[-Q - \frac{Q+1}{\alpha} \ln \left(1 - \alpha \frac{Q}{Q+1} \right) \right] & \text{for } \alpha \leq \alpha_{th}, \\ \zeta_s \frac{(\alpha - 1) Q - 1}{\alpha} & \text{for } \alpha > \alpha_{th}. \end{cases}$$

Recall

$$\alpha_{th} = 1 + 1/Q(\alpha_{th}), \qquad Q(\alpha) = \exp\left(\frac{1+\alpha}{\alpha} R \right) - 1.$$


Outage probability
SNR [dB]
1
10-1
10-2
10-3
10-4
10-5
10-6
-9
-6
-3
0
3
6
9
12
15
18
21
24
α = 0.2, ML
α = 0.2, ML approx.
α = 0.2, LMMSE
α = 0.2, LMMSE approx.
α = 8, ML
α = 8, ML approx.
α = 8, LMMSE
α = 8, LMMSE approx.



L = 1
L = 2
L = 3
Thresholds
Outage gain
1
10-1
10-2
10-3
0
2
4
6
8
10
12
α

- For $\alpha \le \alpha_{th}$ both strategies achieve diversity order $d = 2$ but

$$\kappa = \begin{cases} \kappa_1 \mathbb{1}\{\alpha \le 1\} + \kappa_2 \mathbb{1}\{\alpha > 1\} & \text{AF}, \\ \frac{\zeta_s \zeta_u}{z} Q^2 + \zeta_s \zeta_d (Q+1) \left[-Q - \frac{Q+1}{\alpha} \ln\left(1 - \alpha \frac{Q}{Q+1}\right)\right] & \text{DF}, \end{cases}$$

where

$$\kappa_1 = \frac{\zeta_s \zeta_u}{z} \frac{Q^2}{2} + \zeta_s \zeta_d (Q+1) \left[-Q - \frac{Q+1}{\alpha} \ln\left(1 - \alpha \frac{Q}{Q+1}\right)\right];$$

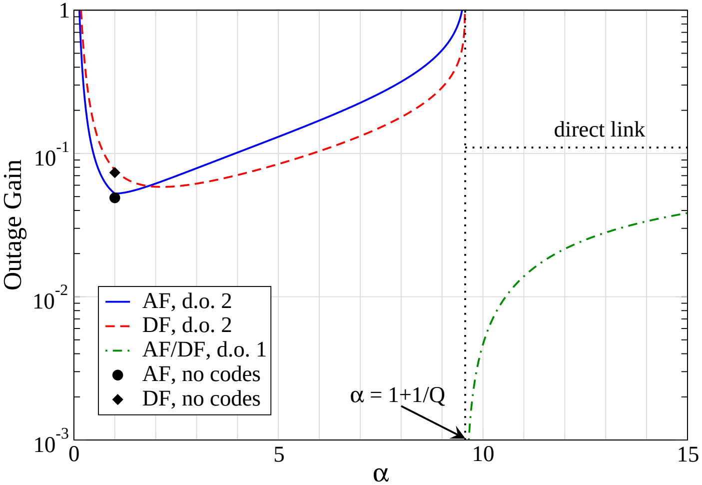
$$\kappa_2 = \frac{\zeta_s \zeta_u}{z} \left\{ \frac{Q^2}{2} + \frac{\alpha - 1}{\alpha} (Q+1) \left[-Q - \frac{Q+1}{\alpha} \ln\left(1 - \alpha \frac{Q}{Q+1}\right)\right]\right\} + \\ + \zeta_s \zeta_d (Q+1) \left[-Q - \frac{Q+1}{\alpha} \ln\left(1 - \alpha \frac{Q}{Q+1}\right)\right].$$

- For $\alpha > \alpha_{th}$ both strategies achieve diversity order $d = 1$ and

$$\kappa = \zeta_s \frac{(\alpha - 1)Q - 1}{\alpha}.$$

AF vs. DF comparison

The 1-relay, LMMSE-receiver case



Conclusions

- Random linear STBC's have been introduced for their simplicity and flexibility.
- Depending on their length and on the receiver, random STBC's give promising results in terms of outage probability and diversity.
- Some details are missing to conclude the analysis of the presented systems (e.g. LMMSE receiver with a generic number of AF relays, approximations for the outage gains, etc.).
- It would be interesting to consider also systems where the source transmits continuously, without knowledge of the presence of the relays.

Recent publications

- D. Gregoratti and X. Mestre, “Random DS/CDMA for the amplify and forward relay channel,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 1017–1027, Feb. 2009.
- ——, “Decode and forward relays: Full diversity with randomized distributed space-time coding,” in *Proc. IEEE ISIT 2009*, Seoul, Korea, Jun. 28–Jul. 3 2009, to be published.
- ——, “Diversity order for the amplify-and-forward multiple-relay channel with randomized distributed space-time coding,” in *Proc. EUSIPCO 2009*, Glasgow, Scotland, Aug. 24–28 2009, to be published.
- ——, “Diversity analysis of a randomized distributed space-time coding in an amplify and forward relay channel,” in *Proc. IEEE ICC 2009*, Dresden, Germany, Jun. 14–18 2009, to be published.
- ——, “The single relay channel: Does randomized coding increase diversity?” in *Proc. ICT-MobileSummit 2009*, Santander, Spain, Jun. 10–12 2009, to be published.